

## Physics Lecture 20 - Pushing the Speed Limit

### Introduction

The WIRL is the Woodcar Independent Racing League, a group of pinewood derby enthusiasts that build and race cars (an addictive hobby). WIRL communicate on the forum at this [link](#). The popular commercially available aluminum track made by BestTrack™ is the one used by the WIRL participants. The purpose of this lecture is to:

- 1) Provide precision BestTrack measurements so the track may be used in the [Virtual Race](#).
- 2) Run a Lima, Ohio WIRL virtual race using a virtual car based on the 12 parameters of the winning WIRL car built by Kevin of Derby Works. Show the power of the [Virtual Race](#) program in helping design faster cars.
- 3) Compare the results of the virtual race with the actual race at Lima and look at ways to push the speed limit.
- 4) Throw in some straightforward but interesting physics now and then (mostly in the boxes).

### Significant Figures

When a PWD car is dropped from a height  $h$ , and we neglect friction, air drag, etc., then we find that its velocity  $v$  after dropping distance  $h$  is  $v = \sqrt{2gh}$ . On the earth, the gravitational constant  $g$  is close to  $980.00 \text{ cm/sec}^2$  and the height  $h$  of the car center of mass start position relative to the level of the finish line is about  $4 \text{ ft} = 48.000" = 121.92 \text{ cm}$ . So get out your calculator and multiply  $2.0000 \times 980.00 \times 121.92$  and then hit the square root key and you get  $488.84 \text{ cm/sec}$ . Because  $12 \text{ inches} = 30.480 \text{ cm}$ , when you divide  $488.84$  by  $30.480$ , then you see that such a velocity is equivalent to  $16 \text{ feet per second}$  (actually,  $16.038$ ). A good ratio to remember is that  $88 \text{ feet per second}$  is exactly  $60 \text{ mph}$ , so scale  $16$  down by the ratio  $60/88$  and you get very close to  $11 \text{ mph}$  (actually,  $10.935 \text{ mph}$ ).

Note that the car wheel rotates about 50 times per second at this speed which is 3000 rpm.

Again, this is the velocity of your car as it crosses the finish line. You get this velocity no matter where the finish line is (as long as it is  $4 \text{ ft}$  lower than the start position). You could trip and accidentally drop the car straight down  $4 \text{ ft}$  to the floor and its center of mass still hits at a velocity of  $488.84 \text{ cm/sec}$ . If your car is  $7 \text{ inches}$  long, or  $7.0000 \times 2.5400 = 17.780 \text{ cm}$  long, then divide by its velocity and you get  $0.03637$  seconds it takes for the whole car length to pass the finish line. Many race timers can see  $0.0001$  seconds, and in this length of time the fraction  $0.0001/0.03637 = 0.002750$  of a car length goes by. This is about  $0.020$  inches or the thickness of 2 playing cards.

The point of all this significant figures stuff is to realize that because of the  $0.0001$  seconds high accuracy of the race timers, we need at least about 4 or 5 significant figures in all the arithmetic, in related measurements like above, if we want to use them to calculate relative finish positions to a small fraction of a car length (It's the old garbage in/garbage out cliché). We used mostly 5 significant figures above, but it is difficult to measure some things like  $h$  to better than about 4 significant figures, so 4 figures is an absolute minimum to be used in what follows.

### Editing in the BestTrack Measurements

The gravitational constant of earth parameter, known as  $g$  or  $G$  (don't confuse with  $g$  for grams), is  $979.27 \text{ cm per second squared}$  for the Houston area. This is known as "The local acceleration of gravity".  $G$  can have different values for different planets depending on their mass. For cities on the earth with various latitudes, the  $G$  value can vary up to about 1%. Multiply the closest latitude (such as 38) by  $0.084$  and add to  $976.83$ . This will give a reasonably accurate value for  $g$  to edit into your track design. When this is done for Houston, at about  $29.8$  latitude, you get  $979.33$ , very close to its accepted value of  $979.27$ . **Table 1** shows a list of  $G$  values that covers the latitudes of most cities in the US. At Lima, Ohio, we have a latitude of  $40.74$  which gives  $g = 980.25$  from **Table 1**.

Latitude	g (cm/sec <sup>2</sup> )
25	978.93
26	979.01
27	979.10
28	979.18
29	979.27
30	979.35
31	979.43
32	979.52
33	979.60
34	979.69
35	979.77
36	979.85
37	979.94
38	980.02
39	980.11
40	980.19
41	980.27
42	980.36
43	980.44
44	980.53
45	980.61
46	980.69
47	980.78
48	980.86
49	980.95
50	981.03

The BestTrack is close to the straight inclined plane track provided as an option in the Virtual Race (VR) program. However, because the BestTrack uses a curved section rather than an abrupt angle where the ramp joins the horizontal run, it is necessary to put in a curvature correction factor into the measurements before they are entered into the VR program. **Figure 1** shows a photo of the BestTrack. **Figure 2** shows the track side view with the track thickness and the extent of the curve greatly exaggerated. An ordinary all straight inclined plane ramp will join the level run at an abrupt angle, as indicated by the heavy dashed lines. But the BestTrack ramp actually has a 4 ft radius curved joining section. Actual measurements show that the height difference **H** (or *h*) between the track below the center of the car and the top surface of the level run is 45.901" (116.59 cm). The distance called **S1** from the car center to the center of the curved transition section (actually the intersection of the heavy dashed lines) is by measurement 101.47". The distance **D**, which is the projection of the straight ramp onto the floor, is  $\sqrt{S1^2 - H^2} = 90.493"$  (229.85 cm). The other distance we need is the horizontal run distance to the center of the optical sensor hole at the finish line. This distance, called SH, is 264.16" on the 35 ft BestTrack. For BestTrack's 42 ft model, add one exactly 7 ft (84") section for a horizontal run distance of SH = 348.16". However, the BestTrack curved path (shown in red) is 2.69" shorter than the normal dashed line inclined plane used in VR, so to compensate for this "short cut" SH must be reduced by this amount, giving 261.47" (664.13 cm) and 345.47" (887.49 cm) for the two BestTrack effective horizontal runs.



Figure 1 - The Aluminum Best Track as presented in their website [BestTrack.com](http://BestTrack.com)

A small piece of physics worth mentioning is that an object traveling at 16 ft/s on a curved path of 4 ft radius is subject to a centripetal acceleration of  $v^2/r = 16^2/4 = 64 \text{ ft/s}^2$ . The acceleration of gravity is  $32 \text{ ft/s}^2$  so there are  $2g$ 's of centripetal acceleration pushing down on the object. To this add the  $1g$  gravitational acceleration to get a  $3g$  total. So a 5 oz car will momentarily weigh 15 oz (almost 1 lb) during the curve travel. This can affect car design such as rear axle strength.

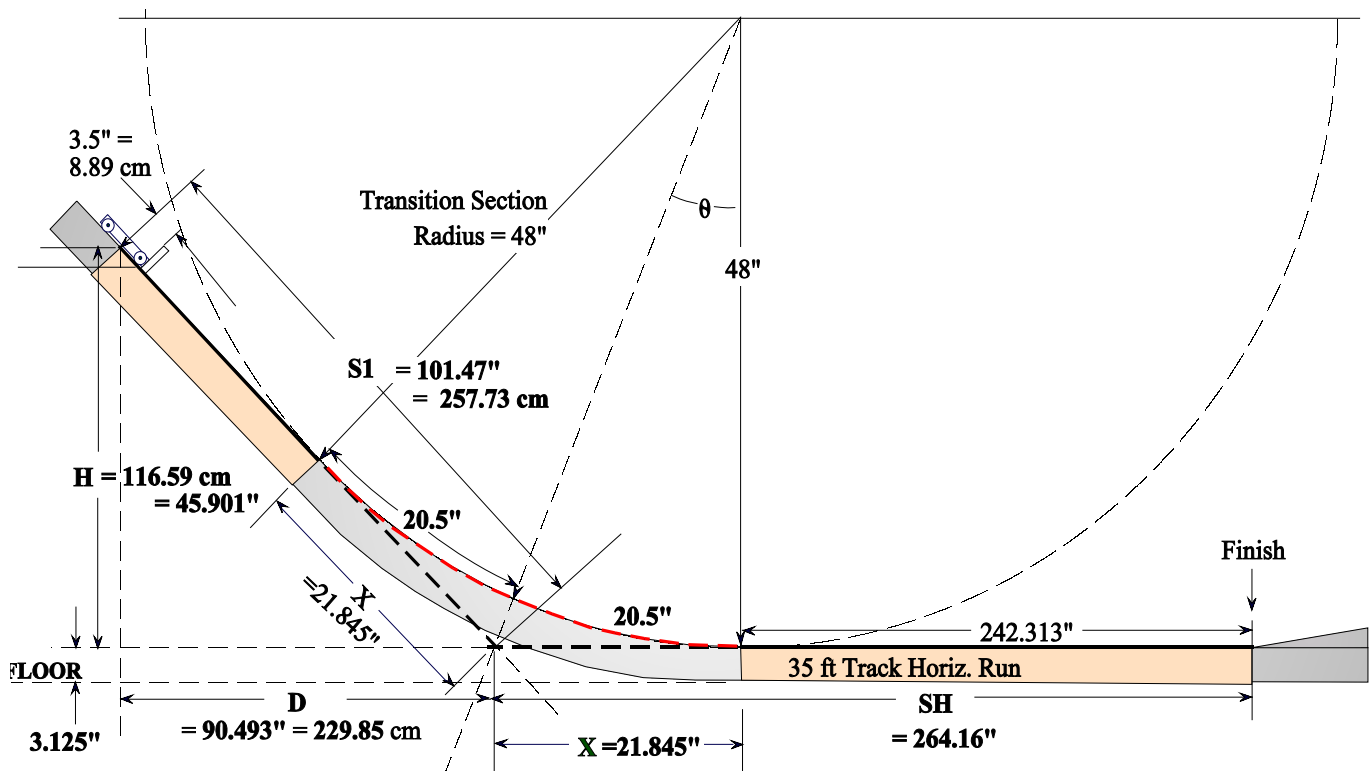


Figure 2 - The Best Track dimension measurements —the extent of the curve and track thickness are exaggerated.

Concerning air density, **Table 2** has been compiled from interpolated values based on density data published in the 1965 version (44<sup>th</sup> Ed) of the Handbook of Chemistry and Physics by the Chemical Rubber Co. Lima, Ohio, has an altitude of 974 ft which gives  $\rho = 0.001129$  g / cm<sup>3</sup> from **Table 2** at 80° F (and RH = 50%). From **Table 2** you can see that air density increases about 0.5% as the temperature goes down 20° F from 80° F to 60° F. And as far as relative humidity is concerned, if you have a wet 100% RH day the air density will only drop about 0.5% and if you have a desert condition of 0% RH the air density would increase about 0.5%.

Most people don't appreciate how heavy air is. If you bought a big box of paper towel rolls at Sam's the empty cardboard box might measure about 39" on a side. This box is about a cubic meter, which is a million cubic centimeters, and **Table 2** shows the air in the box could weigh 1219 grams = 1.219 Kg = 2.7 lbs. Well, that's about what the empty cardboard box itself weighs.

**Table 2 - Air density for various altitudes at 2 temperatures at 50% relative humidity.**

Altitude (ft)	$\rho$ (g / cm <sup>3</sup> )	
	60° F	80° F
0	0.001219	0.001170
500	0.001197	0.001149
1000	0.001176	0.001128
1500	0.001154	0.001107
2000	0.001133	0.001086
2500	0.001111	0.001065
3000	0.001090	0.001044
3500	0.001068	0.001023
4000	0.001046	0.001002
4500	0.001025	0.000981
5000	0.001003	0.000960
5500	0.000982	0.000939
6000	0.000960	0.000918

**Figure 3** shows the key information, from that so far presented (in red), entered into the Track Parameter edit dialog box from the Virtual Race Program. The values of ramp angle ALPHA and the inclined plane length S1 are calculated from the data entered at the top of this dialog box.

**Editing in the Car Measurements**

**Figure 4** shows a photo of the winning car at Lima in the WIRL modified division. The car is a hollowed out piece of balsa wood with several 1/4" tungsten blocks glued in the rear of the hollow shell. The car plan view is shown in **Figure 5**. Kevin (the builder) was kind enough to show me the underside of the car. It was estimated that about 25 of the tungsten blocks, each weighing 4.80 grams, were used for rear weighting. **Table 3** shows the weight of the other components required to get close to 5.0000 oz (141.75 grams).



**Figure 4 - TOPGUN, the winning modified car by Kevin of Derby Works.**

The next thing to do is to enter the 12 parameters of TOP GUN into the Virtual Race program and see how well we can predict its performance. The 12 parameters that we use in a virtual car are explained in [Lecture 10](#). After we create the virtual version of Top Gun we can then discuss what could be done to perhaps increase the speed of the car. The Virtual Race program can then be used to quantify such potential design improvements.

**Track Parameters**

Track name: BestTrack\_WIRL\_LIMA\_20

Switch from cgs to mks (SI):

Density of air (RHO) 0.001129 g/cm<sup>3</sup>

Acceleration of gravity ( G ) 980.25 cm/sec<sup>2</sup>

Length of horizontal run (SH) 877.49 cm

Start height of ramp above horizontal ( H ) 116.59 cm

Projected length of ramp on horizontal ( D ) 229.85 cm

Use bumper time (UBT)

Type of ramp: Inclined plane ramp

Circular arc ramp track starting angle (Y0) 0.938852 radians

Radius of curvature of circ. arc ramp (LP) 283.352 cm

Angle of inclined plane ramp (ALPHA) 0.469426 radians

Length of inclined plane ramp (S1) 257.729 cm

Include transition time (ITT) for inclined plane ramp

Cancel Save

**Figure 3 - The track edit box from the Virtual Race. The Best Track Data is as set up in Lima, Ohio according to **Figure 2** with  $\rho$  and G as explained in the text.**

**Table 3 - Estimated weights for Top Gun**

Part	Weight (g)
25 Tungsten 1/4" cubes	120.00
4 RS Wheels	4.60
4 Axles	0.58
Paint + Glue	3.00
Balsa body	14.50

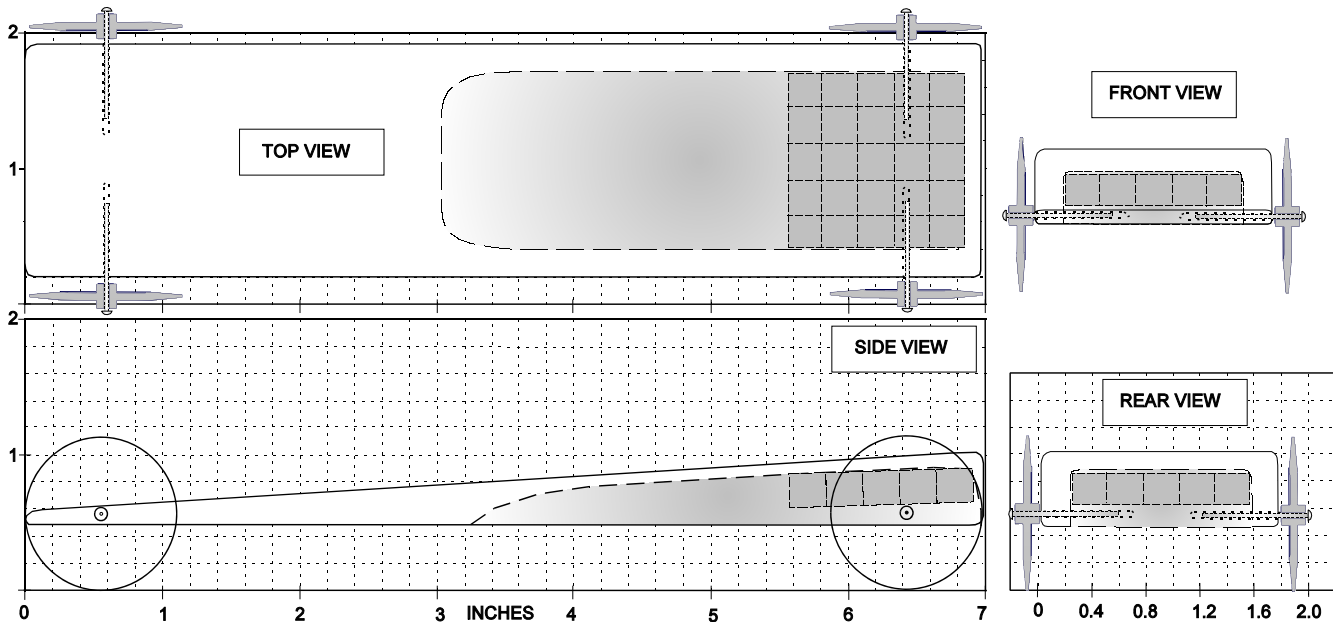


Figure 5 - Plan view of the TOP GUN car by Kevin of Derby Works. Some detail is approximate.

Figure 6 shows the parameters of Top Gun entered into the Virtual Race (VR) Car Parameters dialog box. The parameters are:

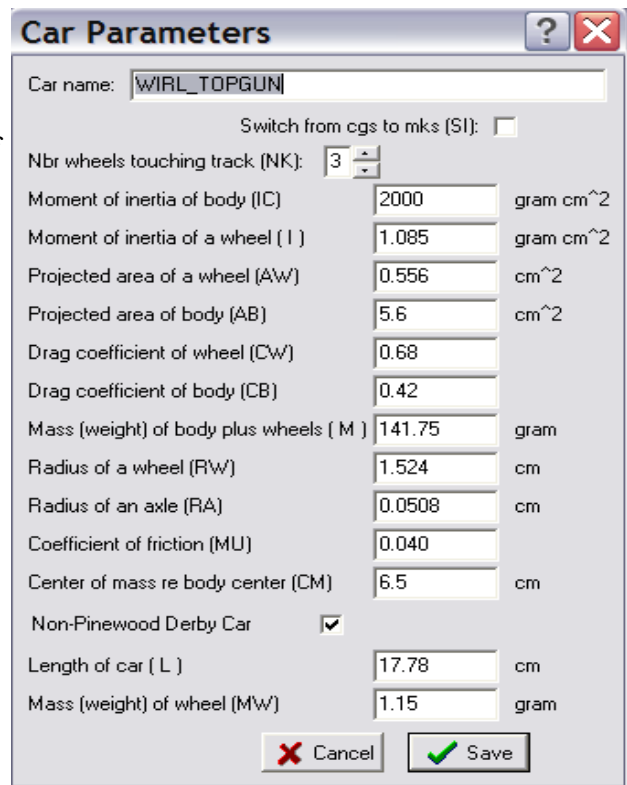


Figure 6 - The Top Gun car parameters edit box from the Virtual Race. The BestTrack Data is as set up in Lima, Ohio according to Figure 2 with  $p$  and  $G$  as explained in the text.

**IC = 2000** is always set at this number and is only used by VR if a car is run on an inclined plane ramp that has an abrupt car rotation where the ramp makes a sharp angle with the straight run. Like the dashed lines of Fig. 2. These type tracks are now rare.

**NK = 3** means that one front wheel is raised so only 3 roll.

**I = 1.085 g-cm<sup>2</sup>** is the moment of inertia of a RS racing wheel.

**AW = 0.556 cm<sup>2</sup>** is the projected area of a RS wheel

**AB = 5.6 cm<sup>2</sup>** is the area of the largest part of the body. In this case, it is right at the rear because of the wedge shape. The body height, 1/2", times the usual 1.75" width gives 0.875 in<sup>2</sup> = 5.6 cm<sup>2</sup>.

**CW = 0.68** is the drag coefficient of a thin wheel such as the RS wheel and the drag coefficient of the body is **CB = 0.42**. These values are discussed in some detail in [Lecture 9](#).

**M = 141.75 grams** is the body mass of close to 5.000 oz.

**RW = 1.524 cm** is the wheel radius for the RS wheel.

**RA = 0.0508 cm** is actually the bore radius of the optional RS wheels which have a 0.040" diameter bore for use with a 0.035" diameter axle.

**MU = 0.040** is the coefficient of friction, and this value is about the smallest that has ever been observed for a PWD wheel/axle. See [Lecture 13](#) where a MU = 0.040 was obtained.

**CM = 6.5 cm** is pretty far back for a center of mass. The rear axle is 0.60" from the rear of the car, and is thus 2.90" from the center of the car where the CM has a reference value of 0. Since 6.5 cm = 2.55", this CM would be 2.90 - 2.55 = 0.35" in front of the rear axle.

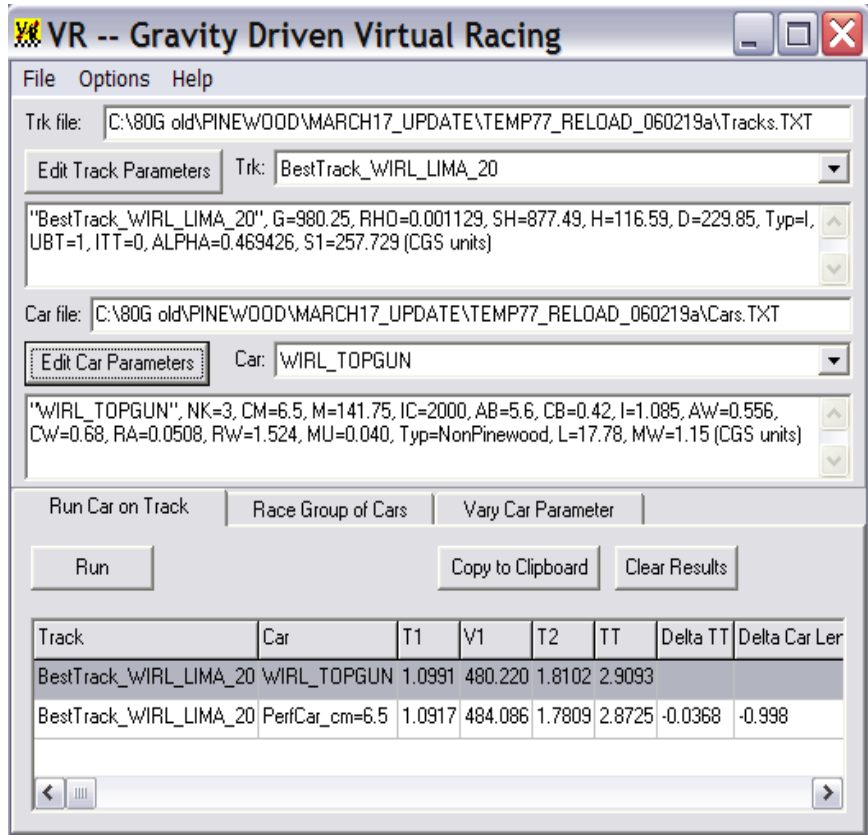
**L = 17.78 cm** -The non-pinewood derby box is checked to allow the individual car length and wheel mass **MW = 1.15 g** to be entered. Normally, for ordinary BSA type stock cars, the length is fixed and the wheel mass is calculated internally by VR from the wheel inertia I. But with thin wheels, this calculation is not accurate and the actual mass should be entered.



### Running the Race

As you can see from **Figure 7**, when we hit [Run] in VR we see a time of 2.9093 s predicted for Top Gun for a perfect race (a perfect race means no energy lost from rubbing or other bumping of the center guide strip). **Table 4** shows the actual race times turned in by Top Gun at Lima. So, in reality, assuming Run 1 was made without energy-costing center strip interference, we see pretty good agreement, about 0.0023s, between calculated and actual best race times. This is only 0.06 car lengths difference. From **Table 4**, after run 1, it appears Top Gun's times increase somewhat, possibly due to lubrication wearing thin.

The best time, 2.907 actual, (2.909 predicted) is to date the fastest time ever observed on a sanctioned PWD race on this particular type track. There are a lot of questions regarding just how fast a car can be made to go while still staying within weight and dimension limits.



**Figure 7 -** The VR main window showing the Top Gun car should turn in a 2.9093 s time for a perfect race on the Lima track..

Now the 2.907 vs. the 2.909 as mentioned shows pretty good agreement with a small difference in the 4<sup>th</sup> significant figure. This is why we went to all the trouble to get the track specified to at least 4 significant figures. But, experimentally, it is almost impossible to control uncertainties to the extent that we could see an actual time drop from say 2.907 (like Run 1) to 2.906 after we, say, moved the CM back slightly. After all, what we would need to see in this case would be the repeat of a perfect race, which are rare to begin with, where there is no effect of center strip rubbing or changes in lube effectiveness or variations in car staging (position at the start gate). Nevertheless, if we keep running this same car with the only changes being the CM moved back slightly, we will increase the *probability* that a better time like 2.906 will be observed. Now the advantage of the VR is that it can predict this effect of a small change in say the CM position and can return a time change reduction of 0.001, without noise, that will be the best predictor of a time change in the actual observed time (if other influences on the car are kept constant). Ramp height  $h$  is very sensitive. Note that an  $h$ , measured as only 1 millimeter higher, 0.040", and set at 116.69 cm, would also show a VR time change of 0.001 s, going from 2.9093 s to 2.9083 s.

**Table 4. Actual times for Top Gun on the Lima BestTrack**

Run	Time (s)
1	2.907
2	2.909
3	2.914
4	2.918
5	2.915
6	2.918
7	2.915
8	2.914
Avg	2.914

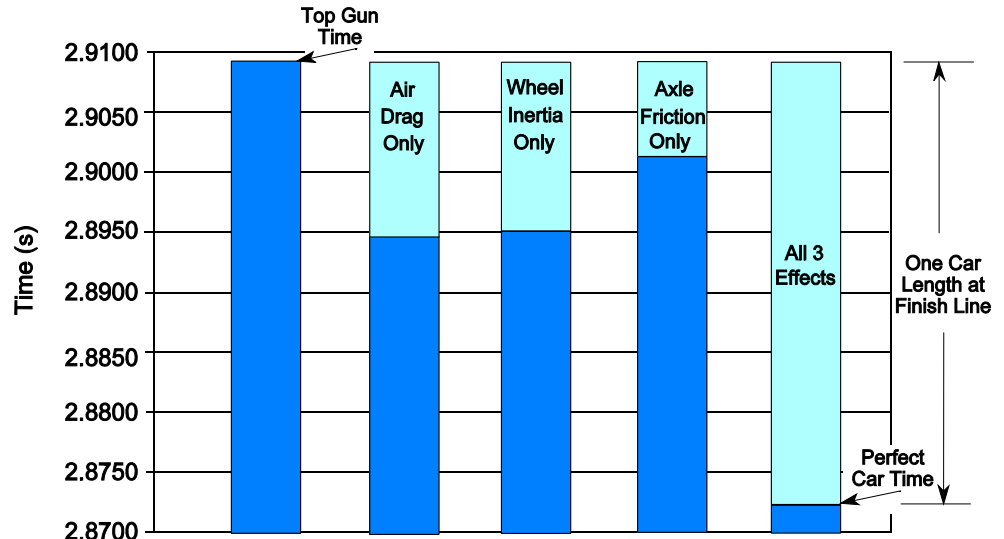
Because of the sensitivity of VR car times to small changes in track parameters, it is recommended to pick a VR track and stick with it for the comparisons of different car designs. There is no need to have absolute agreement with the VR model and real times to within a few milliseconds because relative car times (i.e., time differences) will be representative even if absolute times are somewhat different. Nevertheless, as demonstrated here, you can still get good *absolute* time agreement if you want to spend the effort to make and enter precise track measurements and you can compare to a run that is close to a perfect race. Also, to use a circular arc ramp track like the Piantedosi track, you can use the standard VR track called StdTrk\_HOUSTON\_C\_BT already loaded in the program.

## The Perfect Car

Let's leave the virtual Top Gun CM the same at 6.5 cm and set air drag, wheel/axle friction, and wheel moment of inertia all to zero. We then have a Perfect Car that turns in a 2.8725 s time as shown in the data window in **Figure 7**. So, according to the Virtual Race model, Kevin's Top Gun was within about a car length (0.998) of a car run in a vacuum, with zero friction, and with weightless wheels. Such a perfect car could not actually be built or run, but how close can we get to this 2.8725 s limit? To answer this question, we need to examine the separate effects of wheel inertia, wheel/axle friction, and air drag while keeping the CM constant at 6.5 cm.

### The Effects of Wheel Moment of Inertia, Air Drag, and Wheel/Axle Friction.

As shown in **Figure 8**, the Top Gun time and the Perfect Car time are given as the far left and right dark blue bars. If we run VR with only the body and wheel air drag at zero we get the 2.8947 s time, if we instead set the wheel moment of inertia to zero we get the 2.8954 s time, and if we put this parameter back to its Top Gun value and set wheel/axle friction (MU) at zero we get 2.9013 s. If Top Gun used standard 0.098" bores, friction drag is 2.5 times larger than shown and the time would be 2.9212 s.



**Figure 8** - The effects of air drag, wheel inertia, and axle/wheel friction on Top Gun times.

Next, let's see what might be done to improve the wheel inertia time losses.

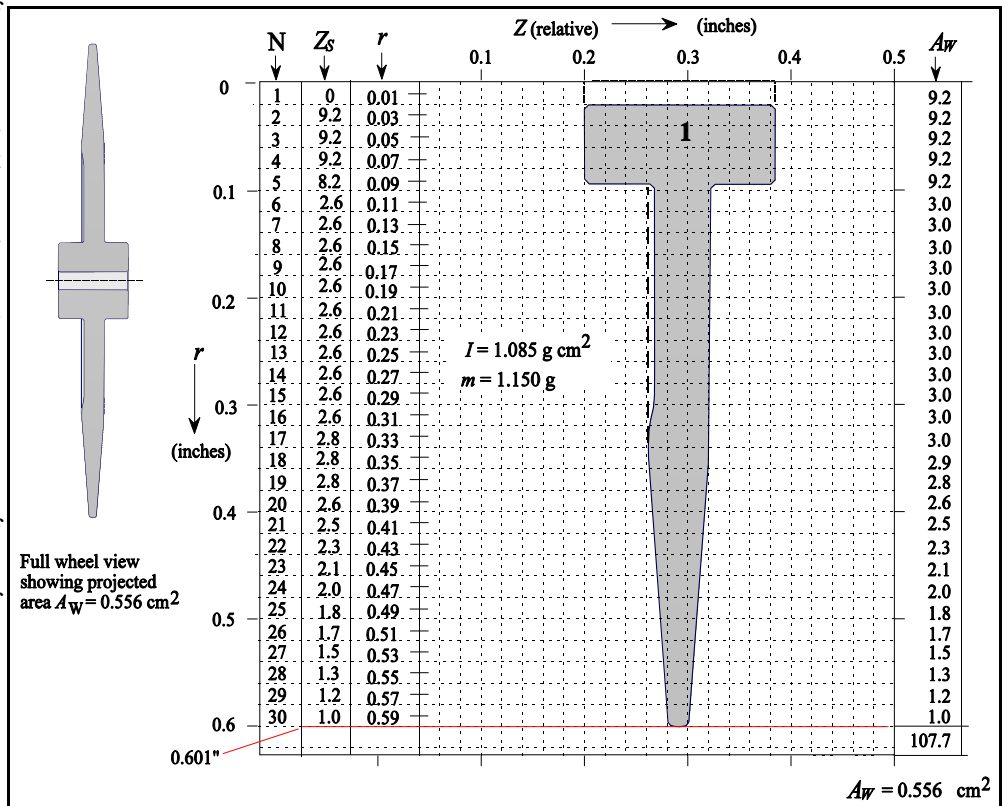
### Wheel Inertia and Related Parameters

When we trim down the RS wheel further to reduce its moment of inertia, we also change its area (AW), its mass (MW), and its radius (RW). Recall when we change the wheel radius, we also change the lever arm that transmits the wheel/axle friction force that acts to slow down the whole car system (see [Lecture 4](#)). So what we will do here is make several virtual wheels, calculate their inertia, area, and mass as shown in [Lecture 5](#). Wheel radius will also be noted and we will assemble these wheels on the Top Gun virtual car to make 6 virtual cars to race. What we are going to demonstrate was first uncovered in 1994 and documented in the large green book [The Physics of the Pinewood Derby](#) which was copyrighted in 2004. Basically, we will now show that the RS wheels as sold are not the optimum size for the fastest runs.

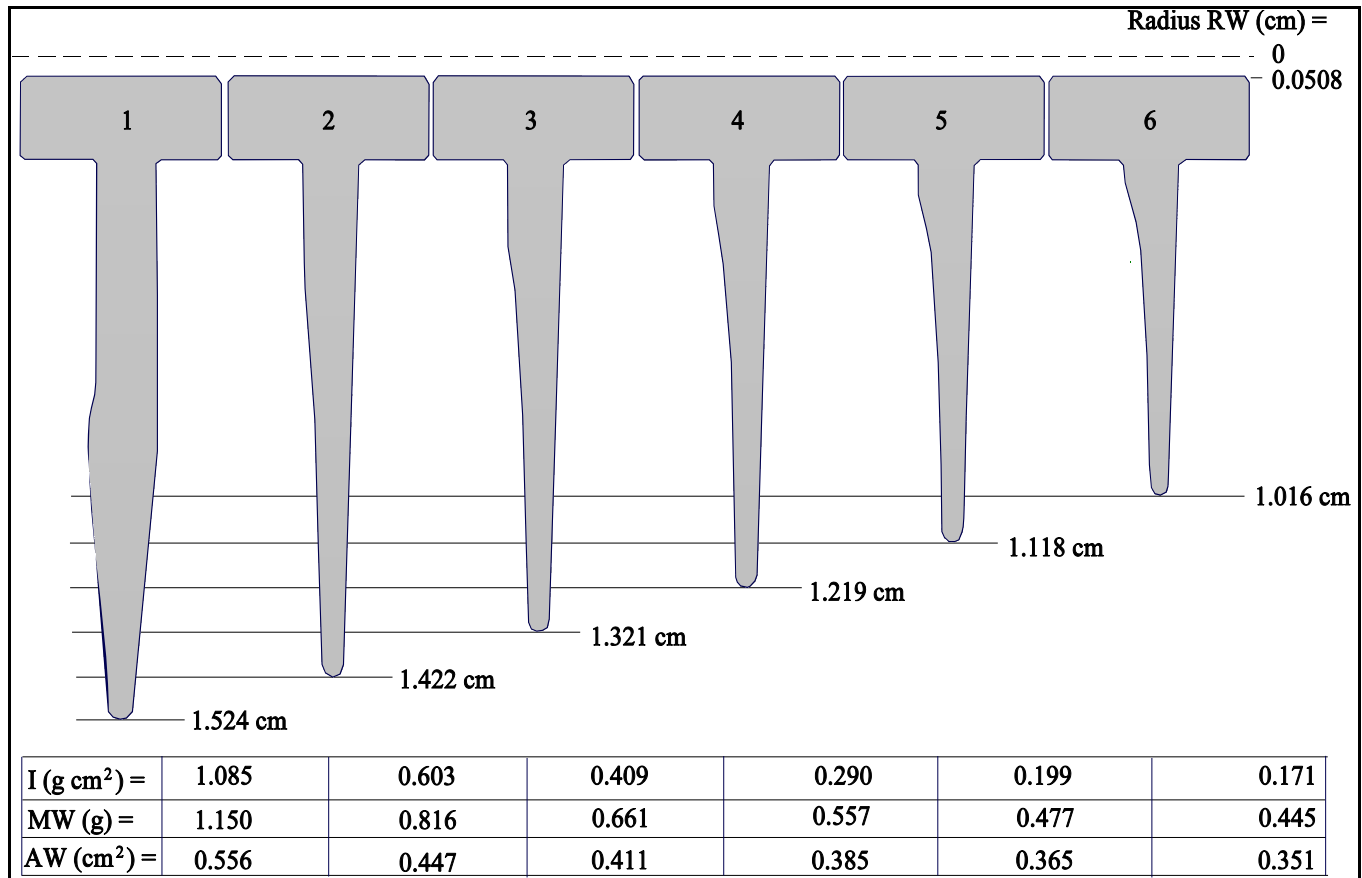
It turns out that when you set up the equations of motion for a gravity driven car and solve the equations you get the time to traverse the given track as a function of the 12 or so parameters of the car. It is not an easy job to solve the equations (a few hundred pages of the [big green book](#) show that the job is fairly involved) but once the solutions are obtained one gets the relationship among the wheel radius, bore radius, coefficient of friction, wheel air drag, and wheel inertia parameters.

Aircraft flight simulators are sophisticated programs that apply appropriate physical and aerodynamic principles in order to make virtual flying seem close to reality. A modern fighter such as the F 22 Raptor is said to require upwards of 6,000 or so parameters in its training and testing simulator. By comparison, the 12 or so parameters required to design and run a virtual PWD car would seem rather simple. Nevertheless, the basic principles used to model real performance are the same in both cases. In the year 2027, a practicing engineer developing models for flights from the moon to Titan, with perhaps 10,000 parameters involved, will recall with a smile his first experience as a Cub Scout with virtual models based on mathematics and natural laws.

**Figure 9** shows the calculation of the moment of inertia of the RS racing wheel according to Lecture 5. Recall in [Lecture 5](#) there was a link to an Excel spreadsheet that would allow a very accurate inertia and mass calculation for a wheel. All one needs to do is estimate how many squares are in the shaded cross section of 1/2 the wheel as shown and list this number in the  $Z_s$  column. Put this column in the spreadsheet and the moment of inertia and mass will appear. Also note to the right of the graph we list the number of  $Z_s$  squares for the projected area of the wheel. Each square is  $0.02" \times 0.02" = 0.508 \times 0.508 = 0.0258 \text{ cm}^2$ . So  $107.7 \text{ squares} \times 2$  (also count the top half of the wheel) gives  $0.556 \text{ cm}^2$  for  $A_w$ . Do this for 5 other wheels of progressively smaller radii and you get the information shown below in **Figure 10**.



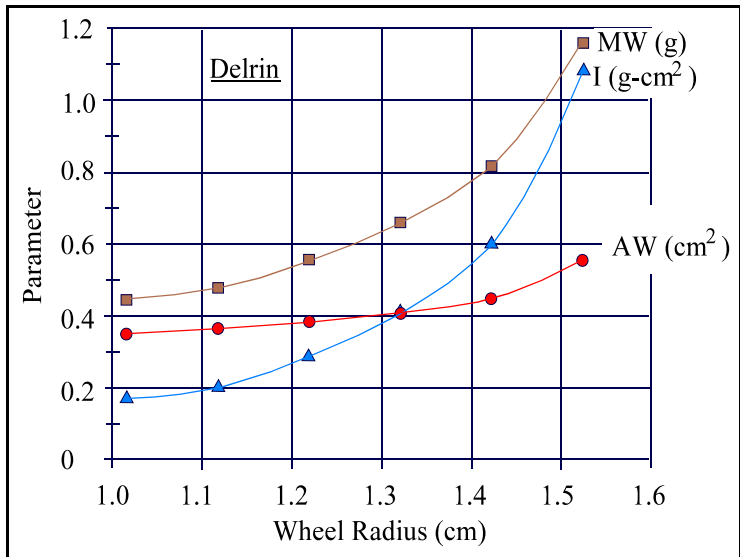
**Figure 9** - The cross section of a RS racing wheel with a 0.040" diameter bore.



**Figure 10** - The results from progressively trimming wheel 1 to create wheels 2 through 6. Mat'l is Delrin.

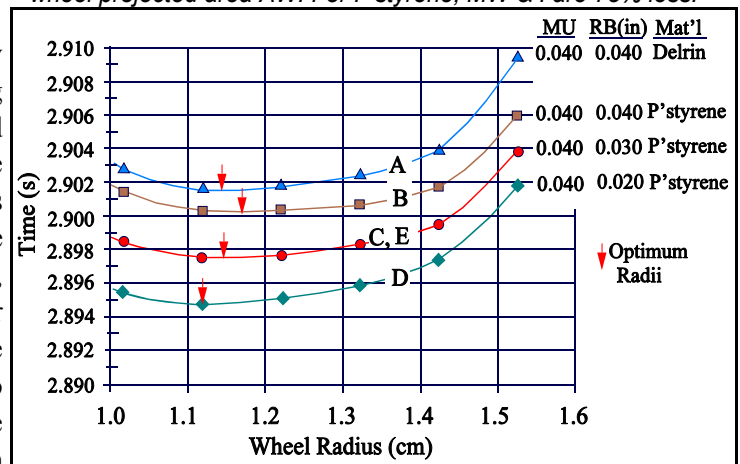
**Figure 8** - The effects of air drag, wheel inertia, and axle/wheel friction on Top Gun times.

**Figure 11** shows the decrease in wheel inertia  $I$  (blue), wheel mass (brown) and wheel cross-sectional or projected area  $AW$  (red) as the radius and outer thickness are progressively reduced. The values of  $I$  and  $AW$  going from wheel 1 to wheel 2 show the largest drop as some extra mass was removed compared to later trimmings.



**Figure 11** - When we analyze the wheels of Figure 10 according to the graphical calculation based on Figure 9, for Delrin we get the values above for the wheel inertia  $I$ , the wheel mass  $MW$ , and the wheel projected area  $AW$ . For P'styrene,  $MW$  &  $I$  are 75% less.

**Figure 12** shows the very interesting results of the virtual races with the different wheels of **Figure 10**. The top right blue triangle marks the time (2.9093 s) we had earlier of Top Gun running Delrin wheels at  $RW = 1.524$  cm radius and  $RB = 0.0508$  cm at a wheel/axle friction of 0.04. Now Delrin™ is a specialized polymer that has good strength properties but has a high density of  $1.40 \text{ g/cm}^3$ . The moment of inertia of a wheel is proportional to the wheel material density, so switching to the ordinary, but still strong, polystyrene wheels will reduce the moment of inertia 75% from 1.085 to 0.814  $\text{g-cm}^2$ . This alone reduces the time to 2.906 s. If we now start to reduce the wheel radii we see two interacting effects, primarily of wheel inertia and transmitted friction. The wheel moment of inertia slows down the overall car CM velocity because it is storing energy as rotation that could be applied to moving the whole vehicle forward. The larger the radius for a given mass, the more inertia the wheel will have. But, on the other hand, the larger the radius, the less is the wheel/axle friction force that will be transmitted to the whole car. So as you reduce the radius and thus  $I$ , there is a point where the frictional time increase starts becoming greater than the time reduction due to smaller  $I$ , and at this point you have found the optimum wheel radius as seen in **Figure 12**.



**Figure 12** - When we run the virtual cars with the wheels of Figure 10, we see where the optimum wheel radius should be depending on wheel material, wheel bore size, and wheel/axle friction.

All classical motion in the universe is either pure translational (all points in a body follows parallel trajectories) or pure rotational (every point in a body describes a circle around a common stationary center) or a combination of the two. A rolling hoop (an all rim wheel) is a combination that has exactly as much rotational energy as it does translational energy (where the CM is the only translating point). So given a fixed amount of potential energy by virtue of raising the whole object in a gravitational field, the rolling hoop at the finish line has a translational velocity of its center given by only  $v = \sqrt{gh}$ . Now if you could get that whole hoop to slide down a ramp of really slick ice of zero friction without rotating its velocity at the finish line would be  $v = \sqrt{2gh}$  or 41.4% greater. A gravity driven car has both rotational and translational components as well, and the less rotational energy that is used the more is left over to increase translational velocity. Perhaps we could explain it to a youngster this way: "Billy, play like you're small or your PWD car is big and you are sitting on the car body between the rear wheels. You are whizzing down the ramp and you peek over the edge and see that the bottom of your left rear wheel is headed backwards away from the finish line at a pretty good clip. As a matter of fact it looks like it's leaving the finish line at the same rate that marks on the track are going backwards. Then you look at the top of the same left rear wheel and it appears it is headed towards the finish line at the same clip—that's good. And you notice the very rear of the wheel is headed straight up and the very front is headed straight down, neither towards the finish line. Billy says, 'Wow, the material in the wheel doesn't seem to know which way the finish line is. I'd probably do better with a sled. At least all the parts would be headed in the right direction.'" Billy is right—with a good sled and a nice no friction slick icy ramp all your energy is translational and you get to the finish line quicker compared to the case where you have to wind up wheels on the way down.



Also, from **Figure 12**, we see that the optimum radius of a RS Delrin racing wheel with a 0.040" bore like on Top Gun is 1.13 cm, considerably smaller than the ones currently on the market (1.524 cm radius). This of course assumes that we have been able to get a MU value of 0.040.

Considering “racing” wheels made from polystyrene, the bottom 3 curves in **Figure 12** are for MU staying constant but the bore size decreasing from 0.040 to 0.030 and on to 0.020". **Table 5** shows the optimum radius gets progressively smaller. At a given RW and thus *I* value,

	Optimum RW (cm)	MU	RB	(MU) X (RB)
B	1.17	0.040	0.040	0.0016
C	1.14	0.040	0.030	0.0012
D	1.11	0.040	0.020	0.0008
Switch MU and RB values of wheels C & E				
E	1.14	0.030	0.040	0.0012

the curves are the same if the friction coefficient MU times the bore radius RB product stays the same. This is because the frictional drag force is proportional to  $MU \frac{RB}{RW}$  We have shown how this works in the bottom line of **Table 5** for

wheel E equivalent to C. Also, we can extrapolate to the more easily obtainable MU value of 0.06 at a bore radius of 0.040" and see that an optimum radius for a polystyrene wheel is 1.23 cm. All the optimum radii so far listed will still easily clear the normal 3/8" high center guide strip clearance test. We should, in the near future, see smaller diameter racing wheels, likely non-Delrin, as soon as this lecture is read and appreciated. Also, the public disclosure prevents someone from trying to patent such wheels. The idea is to spread knowledge and use the PWD to help parents teach kids how physics works.

As far as pushing the speed limit goes, we have by summary that so far:

- Using polystyrene ( $\rho = 1.05 \text{ g/cm}^3$ ) plastic for the RS wheels and trimming them as in **Figure 10** to a radius of 1.17 cm should get us right down to the 2.900 s line.
- There may be an opportunity to also reduce the bore radius (and also the somewhat smaller axle radius) to, say, 0.030". The red line in **Figure 12** shows a perfect race capability of 2.8975 s is possible. Recall the smaller axles on the rear must withstand almost 1 lb force without significant bending or binding during the transition curve.



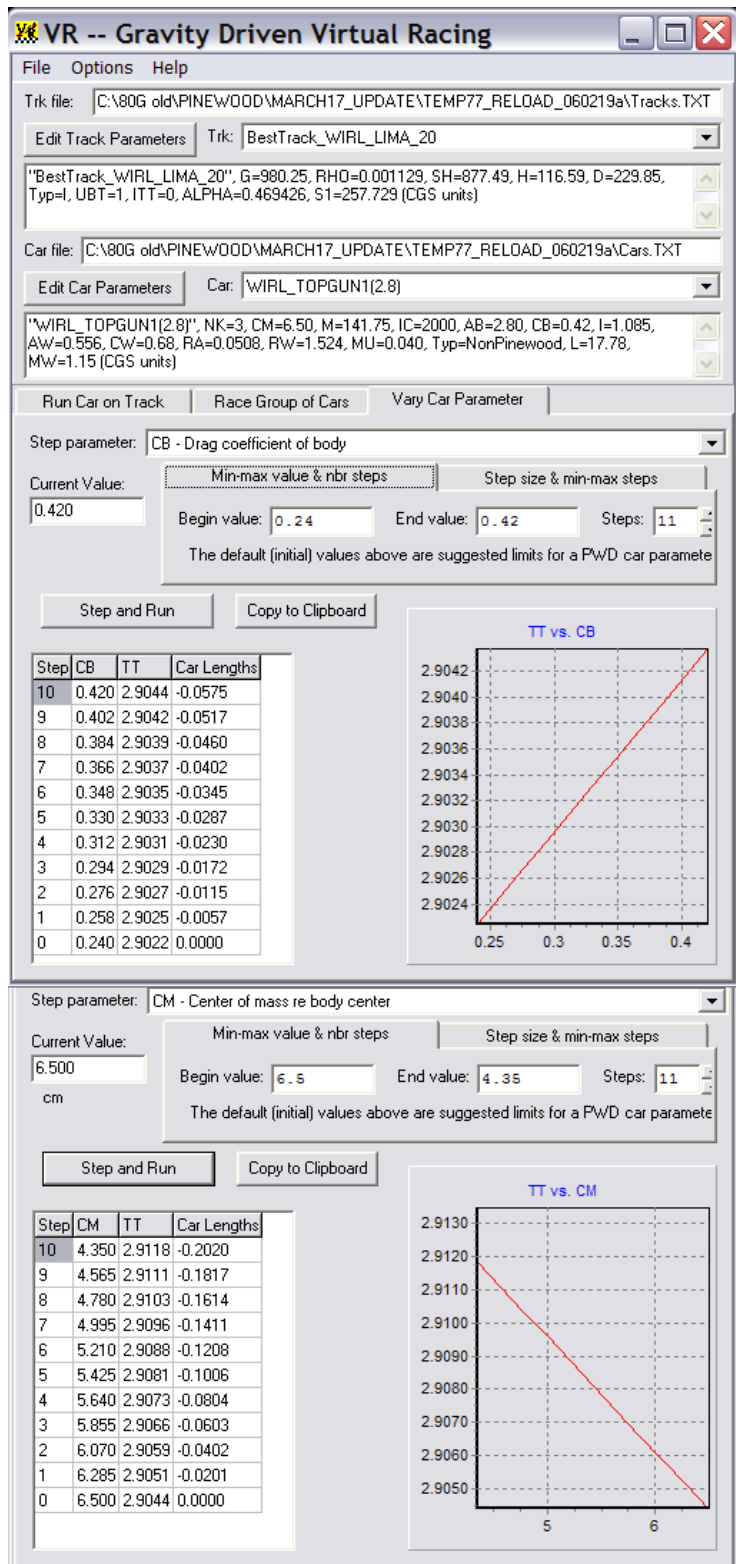
**Figure 13** - Cars from the book *The Physics of the Pinewood Derby*. The center car is also shown in Lecture 4. The left and right cars are pretty fast and have wheels made from regular remolded BSA kit wheels. The times for these cars were predicted closely using the [Virtual Race](#) program.

A somewhat aesthetic point that could be made here is that these Delrin RS racing wheels are made using an expensive (several thousand dollars) computer-controlled CNC lathe which does almost all the work. When non-standard finished parts are mass produced, a valid question might be where do the innovation, creativity, and hands-on skill of the car builder come in? Optimum PWD racing-type wheels of the proper radius composed of standard polystyrene plastic were made and run back in 1993 as documented in the book [The Physics of the Pinewood Derby](#). There it shows how regular polystyrene kit wheels can be heated and remolded to literally any thickness, bore size and diameter to within sub-mil tolerances. For example, in **Fig. 13** the car on the right had thin 2.5" dia. wheels made from kit wheels. The car on the left had optimum radius wheels which were also made from remolded kit wheels. Some districts have rules that allow wheel reshaping provided such wheels use materials only from the official BSA kit. This restriction keeps a level playing field yet still encourages innovation. If the wheels deviate too much from a stock construction, the rules should say the builder is required to race in a Master’s Division race, held concurrently, which is composed of all those who have won trophies in previous years in the ordinary races. Thus one family with access to special tools cannot dominate the regular race year after year. Things like CNC made wheels, that do not use kit materials, are usually allowed only for time tests or in “Outlaw” races.

As we leave the subject of wheel inertia, bore/axle size, wheel/axle friction, wheel material, and wheel outer radius, we might mention that air drag on the thinner and smaller wheels is less which also helps to reduce time slightly. For example, if Top Gun running wheel No. 1 had only its area  $A_W$  change and it went from 0.556 to 0.447  $\text{cm}^2$ , which is the area of wheel No. 2, its time would decrease very slightly from 2.9093 to 2.9084 s, a change of 0.0009 s. However, the  $I$  value going down to that of wheel No. 2 keeping the area and everything else constant gives a change of 2.9093 to 2.9031 s which is a decrease of 0.0062 s, about 7 times more. Also, wheel mass changes have a barely noticeable VR race effect because the overall mass is assumed to be kept constant at 5 oz by adding enough body mass to compensate for that trimmed off the wheels.

### Aerodynamic Improvements

We have basic principles and examples of aerodynamic effects in **Lectures 8, 9 and 10**. Regarding the Top Gun design, we should be able to reduce the maximum height from 0.5" to 0.25", although this would mean the tungsten cubes would be exposed at the top and bottom for lack of space to put wood (epoxy glue works well to form a solid rigid tungsten 1/4" high block of 25 cubes). This would halve  $AB$  to 2.80  $\text{cm}^2$ . Running the VR for this car shows the time drops from 2.9093 to 2.9043, a difference of 0.0050 s. But how about drag coefficient? If we could move the tungsten (and  $CM$ ) forward some we could taper the rear down to a point like an airfoil shape. We looked at this tradeoff in [Lecture 10](#) (see **Fig. 2** there). **Figure 14** shows Top Gun of  $AB = 2.80 \text{ cm}^2$  being run in VR with the  $CB$  reducing from 0.42 to the 0.24 typical of a tapered rear. This is compared to this same Top Gun being run with the  $CM$  going from 6.5 down to 4 cm or so. As you can see, the drag coefficient reduction can gain 0.0022 s but by the time the  $CM$  is moved forward to 5.855 cm you have given the 0.0022 s back. That is just not enough distance to produce a meaningful taper. Of course, as discussed in [Lecture 7](#), on a circular arc ramped track any  $CM$  change will cause about 4 times more time difference than the same  $CM$  change on an inclined plane ramped track. It would help if a tapered tungsten shape, predrilled for axles, could be made for the rear end. Still, we conclude that the drag coefficient cannot easily be changed from 0.42 by rear tapering.



**Figure 14 - The VR results for running [Vary Parameter] for comparing drag coefficient time gain vs CM time loss.**

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### **The Coefficient of Wheel/Axle Friction**

Our original analysis of Top Gun as built was that it must have a MU value close to 0.040. This is a practical lower limit for ordinary sliding friction of two smooth surfaces. Recall in [Lecture 13](#) we were able to get this MU using Super Z Graphite plus Super Z Oil. There is likely no room for coefficient of friction reduction using simple journal bearings.

### **Conclusions**

Using VR to analyze Top Gun improvements suggests we use a polystyrene optimum wheel No 5 of 0.030" bore running, say, a 0.025" nickel plated axle lubed with Super Z Graphite plus Super Z Oil. This should give 2.8975 s for a no bumping race. A reduction of body projected area to 2.8 cm<sup>2</sup> should reduce time further giving 2.8925 s. This is within 0.020 seconds of a Perfect Car and only 0.54 car lengths behind perfection at the finish line, almost twice as close as Top Gun's 1<sup>st</sup> run. But a skilled builder must still control lubrication, rail riding, and car staging strategies to realize this time. Let the race go on.