Physics Lecture 1b - Lets Put Car Weight in Proper Perspective - Part 2

If we lived on an earth-sized planet that didn't have air, then car weight (assuming no friction) wouldn't affect acceleration at all. That's because the gravitational force Mg pulling the car down (weight) would equal the car's inertial resistance to movement Ma so that we just divide out M and see that the car's acceleration $a = g$, where g is a constant everywhere on an airless earth. But if there is air it can push back on any object with an air force proportional to the object's maximum frontal area A times its velocity v squared. There is also more pushing back if the shape factor called drag coefficient C_D is larger or if the air density ρ is larger. So now the gravitational force Mg is reduced by a decelerating force such that

$$Ma = Mg - \frac{1}{2}\rho C_D A v^2$$

$$a = g - \frac{\rho C_D A}{2M} v^2$$

$$a = g - C v^2 \qquad (1)$$

Above we have collected the values of air density, drag coefficient, projected frontal area all divided by 2 times the mass and call it the air drag constant C. The acceleration on the left is the rate of change of velocity with time which equals a constant g less another constant times the velocity squared. Using calculus, equation (1) can be integrated twice to give the following formula for the descent time t of a body falling through air a distance h. The resulting formula is

$$t = \frac{1}{\sqrt{Cg}} \cosh^{-1}(e^{Ch}) \qquad (2)$$

The velocity comes from one integration of the acceleration (1) and is

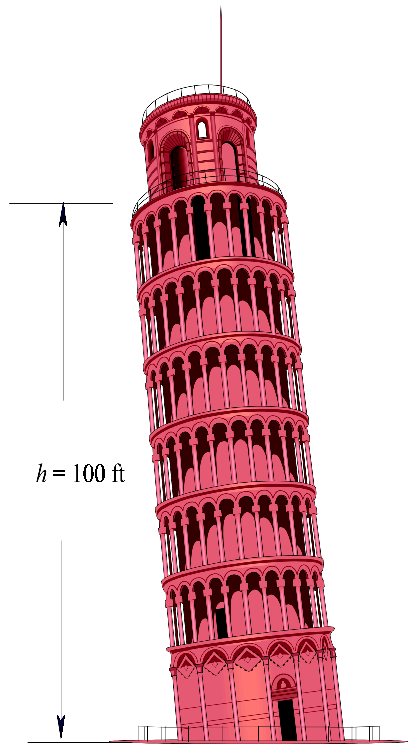
$$v = \sqrt{\frac{g}{C}} \tanh(\sqrt{Cg}\ t) \qquad (3)$$

These functions are available in spreadsheets like EXCEL and **Table 1** shows them applied to falling bodies. They are sometimes called the parachute formulas. Let's suppose Galileo dropped a 5-inch diameter iron ball weighing 7508 grams and a ball about half this weight of diameter 3.94-inch and mass of 3665 grams. If the leaning tower of *pisa* were 100 ft high, then the iron ball weighing half as much hit only 0.00149 s later and only 1.419" behind the heavier ball. Aristotle thought with twice the force pulling down on the heavier object, surely it would hit much quicker than this. But without an electronic timer, it must have looked like a tie. The balls both hit at about 54 mph.

Table 1. *Comparison of impact times of falling bodies in air with Pinewood Derby Cars*

PARAMETERS	COMMON VALUES		TIME DIFF (s)	FINISH DIFF (in)
AIR DENSITY ρ (g cm^3)	0.001225	0.001225		
GRAV CONST g (cm /s^2)	979.27	979.27		
DRAG COEFF. C_D	0.42	0.42		
	LARGE IRON BALL	**SMALL IRON ball**		
DIA (in)	5	3.94		
DIA (cm)	12.7	10.0		
RADIUS (cm)	6.35	5.00		
VOLUME (cm^3)	1072.5	523.60		
AREA A (cm^2)	126.68	78.54		
OBJECT DENSITY (g cm^3)	7.00	7.00		
MASS M (g)	7508	3665		
AIR DRAG CONST C (cm^{-1})	4.34e-06	5.51e-06		
FALL DIST h (ft)	100	100		
FINAL VELOCITY v (mph)	54.295	54.199		
VACUUM FALL TIME t_o (s)	2.49500	2.49500		
AIR FALL TIME t (s)	2.50051	2.50200	0.00149	1.419
	LARGE WOOD BALL	**SMALL WOOD BALL**		
AREA A (cm^2)	126.68	78.5400		
OBJECT DENSITY (g cm^3)	0.70	0.70		
MASS M (g)	751	366.520		
AIR DRAG CONST C (cm^{-1})	4.34e-05	5.51e-05		
FALL DIST h (ft)	100	100		
FINAL VELOCITY v (mph)	51.231	50.371		
VACUUM FALL TIME t_o (s)	2.49500	2.49500		
AIR FALL TIME t (s)	2.55036	2.56539	0.01503	13.327
	PWD CAR - FALL	**PWD CAR - FALL**		
AREA A (cm^2)	18.4	18.4		
MASS M (g)	141.75	140.75		
AIR DRAG CONST C (cm^{-1})	3.34e-05	3.36e-05		
FALL DIST h (ft)	100	100		
FINAL VELOCITY v (mph)	51.988	51.970		
VACUUM FALL TIME t_o (s)	2.49500	2.49500		
AIR FALL TIME t (s)	2.53754	2.53784	0.00030	0.278
	PWD CAR -TRACK	**PWD CAR - TRACK**		
AIR DRAG CONST C (cm^{-1})	5.03e-05	5.06e-05		
FALL DIST h (in)	47	47		
FINAL VELOCITY v (mph)	9.903	9.899		
VACUUM ROLL TIME t_o (s)	2.74620	2.74620		
AIR ROLL TIME t (s)	2.91400	2.91480	0.00080	0.140

Now iron has a density of about 7 g/cm^3, but what would happen if Galileo had used 2 wooden balls of only one-tenth the iron density at 0.7 g/cm^3 ? The air drag constant C is now 10 times larger, meaning that air drag deceleration is now 10 times more than it was. The time difference for impact between balls is now 0.1503 seconds and the lighter ball is 13.37 inches behind when the larger heavier ball hits. Perhaps if Galileo had used wooden balls he would have appreciated air resistance effects more.



Next, let Galileo drop two ordinary pinewood derby cars nose down where one car is 1 gram lighter (about 0.03 oz) so that it is 140.75g (4.97 oz) and the heavier car is 141.75 g (5.00 oz). Play like the cars don't tumble but just fall nose down all the way. They would hit only 0.0003 s apart and their distance separation would be only 0.278" on impact. These cars also have an air drag constant about the same size as the wooden balls.

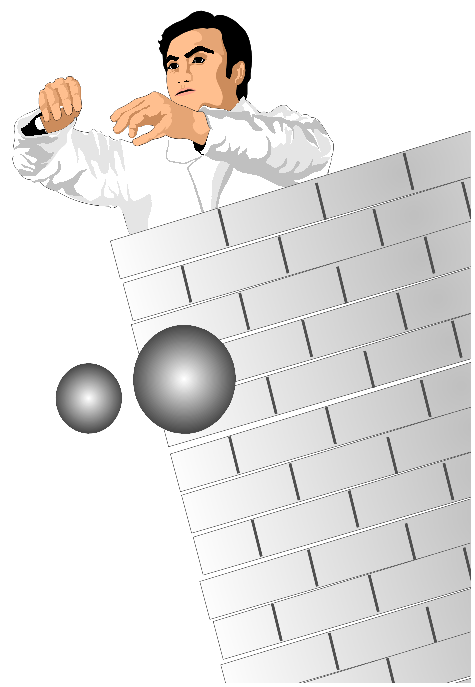
Finally, we let these 2 pinewood derby cars run down a 16 ft ramp from an elevation of 47" and on into a horizontal run of 16 ft to the finish. Here the formulas are a little more complicated than the parachute equation but the Virtual Race has all the mathematics worked out and will give any car time on any track. The VR predicts a time difference of 0.0008 seconds or 0.14" at the finish line. It should be noted that the shape factor or drag coefficient of a sphere is about the same as a pinewood derby car that has a rounded front end and no sharp square corners. Thus C_D = 0.42 for all objects in **Table 1**.

If all the air left the earth and we had only the vacuum of space around us then $Ma = Mg$ and $a = g$, $v = gt$, $h = \frac{1}{2} g t^2$.

From these formulas you can show $v = \sqrt{2gh}$, $t = \sqrt{\dfrac{2h}{g}}$ and you can use the latter to get t_0, the fall time in a vacuum, if you know the drop height h.

Also, knowing that the total initial energy, all potential, is Mgh at height h (where v = 0) , and all kinetic $\frac{1}{2} M v^2$ at the ground (where h = 0) are equal, then divide out M again and get $v = \sqrt{2gh}$

So in summary you see now that with air resistance the mass M just doesn't go away when you divide the equation of motion by it but rather shows up in the denominator of the air resistance term. The more mass M you have the less air resistance can reduce your acceleration a. Now a tiny bit of mass like a fraction of a gram doesn't make a huge difference at the finish line but we see one gram can easily mean the difference between winning and losing because of the high accuracy of the race timing devices. Actually 1 gram means about 1/8" difference at the finish line—and race timers measure 10 times closer than this. And if you are really sloppy with your weight management and go in with a 4 oz car, you can be behind over ½ car length or 4" at the finish line. Thus, to properly manage your car weight, realize:



a) It is about the same importance as friction control (lubrication)

b) It is about the same importance as air drag control (streamlining)

c) The person inspecting your car (weighing) thinks it is a *lot* more important (like Aristotle - Lecture 1a) and he is the judge.

d) Its easy to get maximum weight, compared to optimizing a) or b), so effort-wise its almost free if you have a good scale.

So now the 4 items a), b), c) and d) have put weight in perspective.