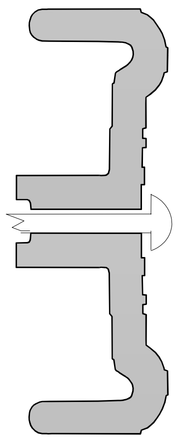
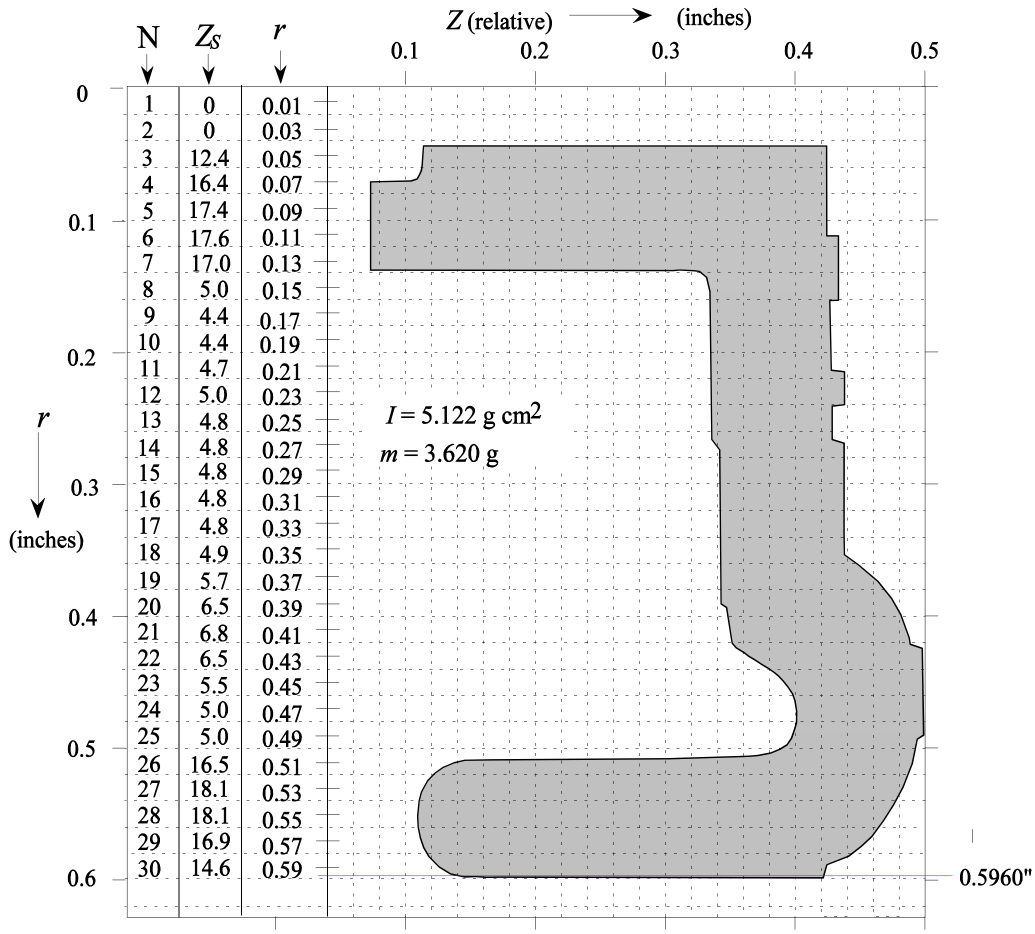
Physics Lecture 5 - How to Calculate the Moment of Inertia of a Wheel



Full Wheel View



N	Z_S	r
1	0	0.01
2	0	0.03
3	12.4	0.05
4	16.4	0.07
5	17.4	0.09
6	17.6	0.11
7	17.0	0.13
8	5.0	0.15
9	4.4	0.17
10	4.4	0.19
11	4.7	0.21
12	5.0	0.23
13	4.8	0.25
14	4.8	0.27
15	4.8	0.29
16	4.8	0.31
17	4.8	0.33
18	4.9	0.35
19	5.7	0.37
20	6.5	0.39
21	6.8	0.41
22	6.5	0.43
23	5.5	0.45
24	5.0	0.47
25	5.0	0.49
26	16.5	0.51
27	18.1	0.53
28	18.1	0.55
29	16.9	0.57
30	14.6	0.59

Figure 1 - 1999 STD WHEEL

Figure 1 above shows a cross-sectional view of a 1999 Standard Wheel as obtained from the kits sold at Scout shops. This particular wheel was introduced in 1999. Accurate dial calipers are used to make measurements for the drawing, sometimes using a wheel that has been sawed in half as shown on the left. The objective is to get the cross-sectional area of 30 wheel "slices" in 0.02" thick radial sections for progressively larger wheel radii *r*. The cross sectional area of a slice is obtained by counting the number of shaded 0.02" x 0.02" squares called Z_S in the *z* direction. For example, the slice centered at radius 0.05" has 15 Z_S squares all shaded about 80% full for 0.8 x 15 =12. The leftmost square looks about 0.4 full so the slice total is 12.4. On the last page, in **Figure 2**, there is a blank grid for you to print and determine the Z_S for your modified wheel. You can put your value for moment of inertia into the virtual race program to see how your finish times are affected. **(** See EXCEL link next page)

Table 1 shows the spread sheet calculation which is straightforward. From scratch, put numbers from **Fig. 1** into columns A, B, and E and use columns D and F to convert from inches to centimeters. C is 0.02 B. Column G is column F cubed. Column H is the thickness of each slice in the *r* direction in centimeters which is (2.54) (0.02"). Now, to add up or "integrate" in the *Z* direction we have column I which is the product of columns D, G, and H. Finally column J does a cumulative sum of entries in column I to "integrate" in the *r* direction and get the result at the bottom right box. Multiply this number by 2π to integrate in a circular direction and then multiply by the density of polystyrene (1.05) to get $I = 5.122$ g cm^2 as the final value. Congratulations! You have just numerically integrated the moment of inertia *I* for a wheel of radius R_W and variable thickness *z* as given by:

$$I = 2\pi\rho \int_0^{R_w} r^3\, dr \int_0^{Z} dz$$

***Table 1**. Moment of inertia calculation for the 1999 Standard Wheel*

N or i	z_s No of squares in z direction, each Δr x z = 0.02" x 0.02"	$\sum \Delta z =$	Z	r_i	r_i	r_i^3	Δr	MOMENT INCREMENT $Z r_i^3 \Delta r$	INTEGRAL $\sum_{i=1}^{i=N} Z r_i^3 \Delta r$
		inch	cm	inch	cm	cm^3	cm		
A	B	C = 0.02 B	D = 2.54 C	E	F = 2.54 E	G = F^3	H = 2.54*0.02	I = D*G*H	J = CUMSUM I
1	0	0	0	0.01	0.0254	0.0000	0.0508	0	0
2	0	0	0	0.03	0.0762	0.0004	0.0508	0	0
3	12.4	0.248	0.6299	0.05	0.1270	0.0020	0.0508	0.0001	0.0001
4	16.4	0.328	0.8331	0.07	0.1778	0.0056	0.0508	0.0002	0.0003
5	17.4	0.348	0.8839	0.09	0.2286	0.0119	0.0508	0.0005	0.0008
6	17.6	0.352	0.8941	0.11	0.2794	0.0218	0.0508	0.0010	0.0018
7	17.0	0.340	0.8636	0.13	0.3302	0.0360	0.0508	0.0016	0.0034
8	5.0	0.100	0.2540	0.15	0.3810	0.0553	0.0508	0.0007	0.0041
9	4.4	0.088	0.2235	0.17	0.4318	0.0805	0.0508	0.0009	0.0050
10	4.4	0.088	0.2235	0.19	0.4826	0.1124	0.0508	0.0013	0.0063
11	4.7	0.094	0.2388	0.21	0.5334	0.1518	0.0508	0.0018	0.0082
12	5.0	0.100	0.2540	0.23	0.5842	0.1994	0.0508	0.0026	0.0107
13	4.8	0.096	0.2438	0.25	0.6350	0.2560	0.0508	0.0032	0.0139
14	4.8	0.096	0.2438	0.27	0.6858	0.3225	0.0508	0.0040	0.0179
15	4.8	0.096	0.2438	0.29	0.7366	0.3997	0.0508	0.0050	0.0228
17	4.8	0.096	0.2438	0.33	0.8382	0.5889	0.0508	0.0073	0.0362
18	4.9	0.098	0.2489	0.35	0.8890	0.7026	0.0508	0.0089	0.0451
19	5.7	0.114	0.2896	0.37	0.9398	0.8301	0.0508	0.0122	0.0573
20	6.5	0.130	0.3302	0.39	0.9906	0.9721	0.0508	0.0163	0.0736
21	6.8	0.136	0.3454	0.41	1.0414	1.1294	0.0508	0.0198	0.0934
22	6.5	0.130	0.3302	0.43	1.0922	1.3029	0.0508	0.0219	0.1153
23	5.5	0.110	0.2794	0.45	1.1430	1.4933	0.0508	0.0212	0.1365
24	5.0	0.100	0.2540	0.47	1.1938	1.7014	0.0508	0.0220	0.1584
25	5.0	0.100	0.2540	0.49	1.2446	1.9279	0.0508	0.0249	0.1833
26	16.5	0.330	0.8382	0.51	1.2954	2.1738	0.0508	0.0926	0.2758
27	18.1	0.362	0.9195	0.53	1.3462	2.4397	0.0508	0.1140	0.3898
28	18.1	0.362	0.9195	0.55	1.3970	2.7264	0.0508	0.1273	0.5171
29	16.9	0.338	0.8585	0.57	1.4478	3.0348	0.0508	0.1324	0.6495
30	14.6	0.292	0.7417	0.59	1.4986	3.3656	0.0508	0.1268	0.7763
MOMENT OF INERTIA = $2\pi\rho(0.7763) = (6.2831)(1.05)(0.7763) = 5.122$ g cm^2									

As a check on the accuracy of the wheel cross section representation, simply change column G to r rather than r^3. The integral, times 2π, now becomes just the volume, which, when multiplied by polystyrene density, gives wheel mass in grams. Thus the lower right hand box changes from 0.7763 to 0.5528 and you get wheel weight as

$$M = 2\pi\rho \int_0^{R_w} r\,dr \int_0^{Z} dz = \rho V = (6.2831)(1.05)(0.5528) = 3.647 \text{ g}$$

This compares to 3.620 g for the average measured weight of several wheels which is 0.7% accuracy.

What you should do: Go to this link to download the EXCEL spreadsheet for the 1999 Standard wheel shown in **Fig. 1**. Then sketch your wheel area in **Fig. 2**, (or modify **Fig 1.)**, estimate your new *Z*s values if different from **Fig 1**., and change the *Zs* column (keeping G column as r^3), and the moment of inertia value for your wheel will appear at the bottom.

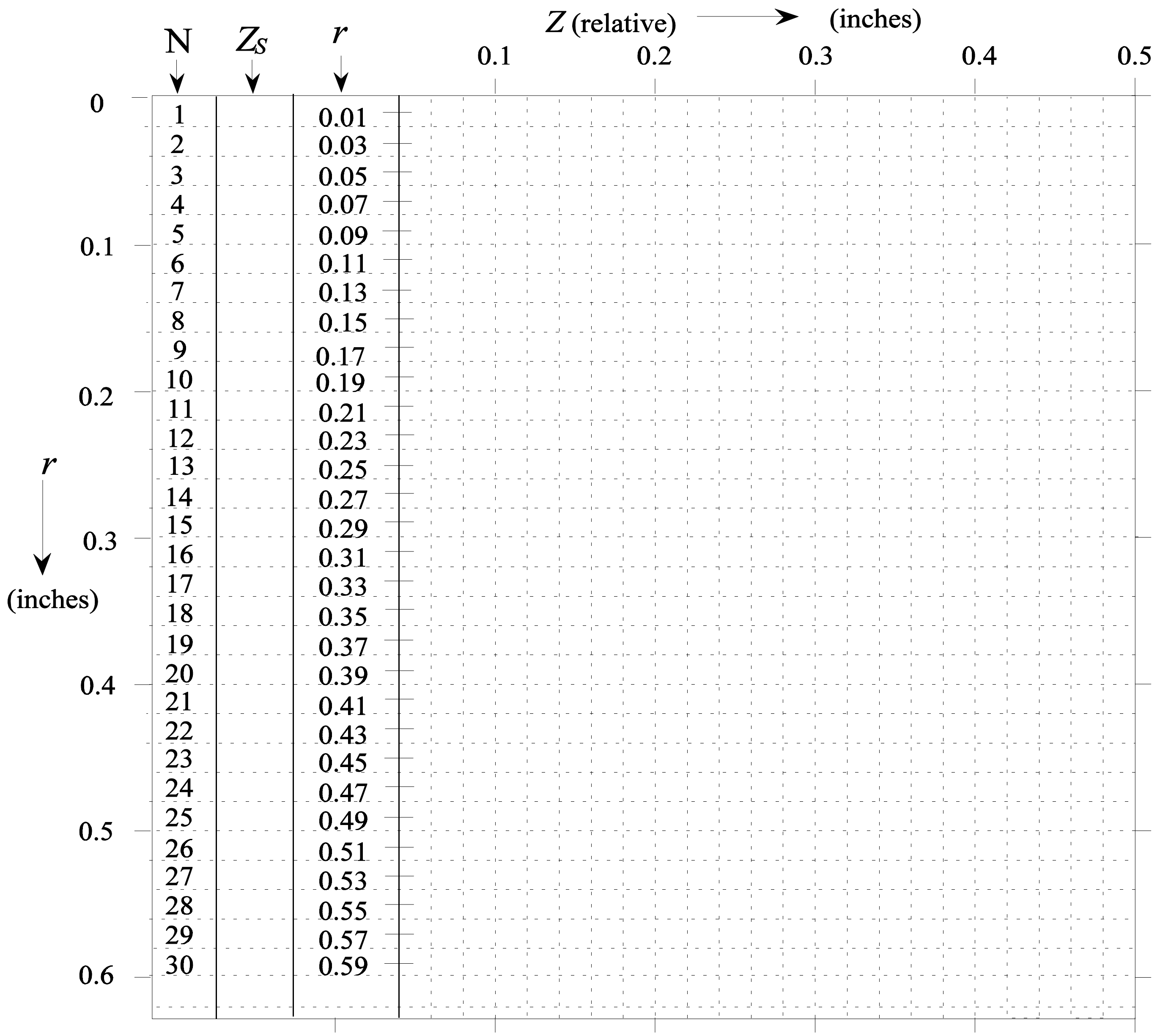


Figure 2 - Blank grid for sketching in your wheel cross section area.